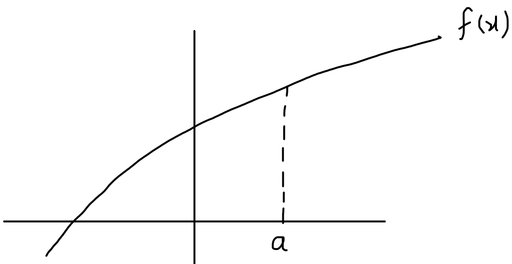
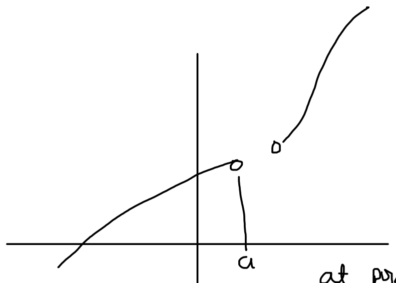
Continuity and Differentiability

Thursday, May 11, 2023 7:08 PM

Continuity





at point a
$x = a$
$L.H.L \neq R.H.L = f(a)$
Here $f(x)$ is dis-con. at $x=a$

at point a
$x=a$
$L.H.L = R.H.L = f(a)$
Here $f(x)$ is Continuous

Q.1 $f(x) = 5x - 3$

at $x = 0$

L.H.L

$\lim_{x \to 0^-} f(x)$

$\lim_{h \to 0} f(0-h)$ → Small value

$\lim_{h \to 0} 5(0-h) - 3$

$\lim_{h \to 0} 5(-h) - 3$

$5(-0) - 3$

$0 - 3$

-3

R.H.L $\lim_{x \to 0^+} f(x)$

$\lim_{h \to 0} f(0+h)$

$\lim_{h \to 0} 5(0+h) - 3$

$\lim_{h \to 0} 5h - 3$

$5 \times 0 - 3$

$0 - 3$

-3

at $x = 0$

$f(0)$

$5 \times 0 - 3$

-3

Since $L.H.L = R.H.L = f(0)$
Hence $f(x)$ is cont. at

$f(x) = 5x - 3$

at $x = 5$

LHL $= \lim_{x \to 5^-} f(x)$

$\lim_{h \to 0} f(5-h)$

$\lim_{h \to 0} 5(5-h) - 3$

$5(5-0) - 3$

$25 - 3$

22

$\{ \; x \text{ if } x \leq 1$

$$f(x) = \begin{cases} x & \text{if } x \le 1 \\ 5 & \text{if } x > 1 \end{cases}$$

at $x = 1$

$$L.H.L = \lim_{x \to 1^-} f(x)$$

$$\lim_{h \to 0} f(\underbrace{1-h}_{x})$$

$$\lim_{h \to 0} 1 - h$$

$$1 - 0$$

$$\underline{\underline{1}}$$

Find all points of discontinuity of f, where f is defined by

6. $f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$

7. $f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$

8. $f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$

9. $f(x) = \begin{cases} \dfrac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \ge 0 \end{cases}$

10. $f(x) = \begin{cases} x+1, & \text{if } x \ge 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$

11. $f(x) = \begin{cases} x^3-3, & \text{if } x \le 2 \\ x^2+1, & \text{if } x > 2 \end{cases}$

12. $f(x) = \begin{cases} x^{10}-1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$

for point $\underline{\underline{x = -3}}$

$$\underline{L.H.L} \quad \lim_{x \to -3^-} f(x)$$

$$\lim_{h \to 0} f(-3-h)$$

$$\lim_{h \to 0} |-3-h| + 3$$

$$|-3-0| + 3$$

$$3 + 3$$

$$\underline{\underline{6}}$$

$$\underline{R.H.L} \quad \lim_{x \to -3^+} f(x)$$

$$\lim_{h \to 0} f(-3+h)$$

$$\lim_{h \to 0} -2(-3+h)$$

$$-2(-3+0)$$

$$\underline{\underline{6}}$$

$$f(-3) = |-3| + 3$$

$$= \underline{\underline{6}}$$

Find the values of k so that the function f is continuous at the indicated point in Exercises 26 to 29.

26. $f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x}, & \text{if } x \ne \dfrac{\pi}{2} \\ 3, & \text{if } x = \dfrac{\pi}{2} \end{cases}$ at $x = \dfrac{\pi}{2}$

27. $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$ at $x = 2$

28. $f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ at $x = \pi$

29. $f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$ at $x = 5$

30. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$

is a continuous function.

<u>Sol. 26</u> Given →

$f(x)$ is continuous at $x = \frac{\pi}{2}$

$$L.H.L = R.H.L = f\left(\frac{\pi}{2}\right) \text{ — } (1)$$

$$\because \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\underline{L.H.L} \quad \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \to 0} \frac{k \sin h}{\cancel{\pi} - \cancel{\pi} + 2h} = \lim_{h \to 0} \frac{k \sin h}{2h} = \frac{k}{2}$$

R.H.L -

Find the values of k so that the function f is continuous at the indicated point in Exercises 26 to 29.

$$\begin{cases} \dfrac{k \cos x}{}, & \text{if } x \ne \dfrac{\pi}{2} \end{cases}$$

Find the values of k so that the function f is continuous at the indicated point in Exercises 26 to 29.

26. $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$ at $x=\dfrac{\pi}{2}$

27. $f(x)=\begin{cases}kx^2, & \text{if } x\leq 2\\ 3, & \text{if } x>2\end{cases}$ at $x=2$

28. $f(x)=\begin{cases}kx+1, & \text{if } x\leq \pi\\ \cos x, & \text{if } x>\pi\end{cases}$ at $x=\pi$ $\rightarrow$

R.H.L

$\lim_{x\to\pi^+} f(x)$

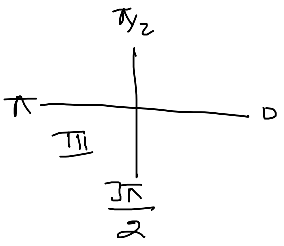
$\lim_{h\to 0} f(\pi+h)$

$\lim_{h\to 0} \cos(\pi+h)$

$\lim_{h\to 0} -\cos h$

$-\cos 0$

$\underline{\underline{-1}}$



29. $f(x)=\begin{cases}kx+1, & \text{if } x\leq 5\\ 3x-5, & \text{if } x>5\end{cases}$ at $x=5$

30. Find the values of a and b such that the function defined by

$$f(x)=\begin{cases}5, & \text{if } x\leq 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\geq 10\end{cases}$$

is a continuous function.

Find the values of k so that the function f is continuous at the indicated point in Exercises 26 to 29.

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is a continuous function.

Differention

① $\dfrac{d}{dx}x^n = nx^{n-1}$

② $\dfrac{d}{dx}x = 1$

③ $\dfrac{d}{dx}\sqrt{x} = \dfrac{1}{2\sqrt{x}}$

④ $\dfrac{d}{dx}K = 0$

⑤ $\dfrac{d}{dx}\left(\dfrac{1}{x^n}\right) = -\dfrac{n}{x^{n+1}}$

Ex $\dfrac{d}{dx}\left(\sqrt{x}-\dfrac{1}{3}x^4-3x+40-\dfrac{1}{x^2}\right)$

$\dfrac{1}{2\sqrt{x}}-\dfrac{4x^3}{3}-3+\dfrac{2}{x^3}$

Ex $\dfrac{d}{dx}\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{2\sqrt{x}}+\dfrac{x^{\frac{3}{2}}}{2}$ ✓

$-\dfrac{1}{x^{\frac{1}{2}}} = -\left(\dfrac{-\frac{1}{2}}{x^{\frac{3}{2}}}\right)$

$= \underline{\underline{\dfrac{1}{2x^{3/2}}}}$

Product Rule

$\frac{d}{dx}(I \cdot II)$

$I \frac{d}{dx} II + II \frac{d}{dx} I$

Qoutient Rule

$\frac{d}{dx}\left(\frac{I}{II}\right)$

$\frac{II \frac{d}{dx} I - I \frac{d}{dx} II}{II^2}$

Chain Rule

$\frac{d}{dx} e^{\square} = e^{\square} \frac{d}{dx} \square$

Ex $\frac{d}{dx}(x^2 \cdot \log x)$

$x^2 \cdot \frac{1}{x} + \log x \cdot 2x$

$x + 2x \cdot \log x$

$x(1 + 2\log x)$

Ex $\frac{d}{dx}\left(\frac{a + b\sin x}{a - b\sin x}\right)$

$\frac{(a - b\sin x)(0 + b\cos x) - (a + b\sin x)(-b\cos x)}{(a - b\sin x)^2}$

$\frac{ab\cos x - \cancel{b^2\sin x\cos x} + ab\cos x + \cancel{b^2\sin x\cos x}}{(a - b\sin x)^2}$

$\frac{2ab\cos x}{(a - b\sin x)^2}$

EXERCISE 5.2

Differentiate the functions with respect to x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$
2. $\cos(\sin x)$
3. $\sin(ax + b)$
4. $\sec(\tan(\sqrt{x}))$
5. $\frac{\sin(ax + b)}{\cos(cx + d)}$
6. $\cos x^3 \cdot \sin^2(x^5)$
7. $2\sqrt{\cot(x^2)}$
8. $\cos(\sqrt{x})$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \cdot \tan x$

$y = \sec(\tan(\sqrt{x}))$

$\frac{dy}{dx} = \sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

Sol. 1 Let $y = \sin(x^2 + 5)$

D.w.r.t. x

$\frac{dy}{dx} = \cos(x^2 + 5) \cdot 2x$

$\cos x^3 = -\sin x^3 \cdot 3x^2$

$(\sin(x^5))^2$

EXERCISE 5.2

Differentiate the functions with respect to x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$
2. $\cos(\sin x)$
3. $\sin(ax + b)$
4. $\sec(\tan(\sqrt{x}))$
5. $\frac{\sin(ax + b)}{\cos(cx + d)}$
6. $\cos x^3 \cdot \sin^2(x^5)$
7. $2\sqrt{\cot(x^2)}$
8. $\cos(\sqrt{x})$

$\boxed{\sin 2x = 2\sin x \cdot \cos x}$

$y = \cos x^3 \cdot (\sin x^5)^2$

$\frac{dy}{dx} = \cos x^3 \left[2\sin x^5 \cdot \cos x^5 \cdot (5x^4)\right] + (\sin x^5)^2 \cdot (-\sin x^3) 3x^2$

$y = 2\sqrt{\cot(x^2)}$

$\boxed{\frac{d}{dx}\cot x = -\csc^2 x}$

dy d 1 (...)

$y = 2\sqrt{\cot(x^2)}$ $\boxed{\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x}$

$\frac{dy}{dx} = \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{\cot(x^2)}} \cdot (-\operatorname{cosec}^2 x^2)\, 2x$

$= \frac{-2x \cdot \operatorname{cosec}^2 x^2}{\sqrt{\cot(x^2)}}$

$= \frac{-2x}{\sin^2 x^2 \cdot \frac{\sqrt{\cos x^2}}{\sqrt{\sin x^2}}}$

$= \frac{-2x}{(\sin x^2)^{\cancel{2}} \cdot \frac{\sqrt{\cos x^2}}{\cancel{(\sin x^2)^{1/2}}}} = \frac{-2x}{(\sin x^2)^{2-\frac{1}{2}} \cdot \sqrt{\cos x^2}} = \frac{-2x}{(\sin x^2)^1 \sqrt{\sin x^2} \cdot \sqrt{\cos x^2}}$

$\boxed{\sin 2x = 2\sin x \cdot \cos x}$

$= \frac{-2x}{\sin x^2 \sqrt{\frac{2\sin x^2 \cdot \cos x^2}{2}}}$ $\sin 2x^2$

$= \frac{-2x\sqrt{2}}{\sin x^2 \sqrt{\sin 2x^2}} = \frac{-2\sqrt{2}x}{\sin x^2 \cdot \sqrt{\sin 2x^2}}$

Ans: $\frac{-2\sqrt{2}\,x}{\sin x^2 \cdot \sqrt{\sin 2x^2}}$

$\frac{dy}{dx} = \cos x^3 \left[2\sin x^5 \cdot \cos x^5 \cdot (5x^4) \right] + (\sin x^5)^2 (-\sin x^3)\, 3x^2$

$= 5x^4 \cos x^3 \cdot \sin 2x^5 - 3x^2 \cdot \sin^2 x^5 \cdot \sin x^3$

Derivative of explicit function.

$y = x^2 + \sin x + \sqrt{x} + e^x$

$(\sin x)^3$

Derivative of Implicit function.

$y = x^2 + y^3 + xy + \sin x + e^y$

$\frac{dy}{dx} = 2x + 3y^2 \frac{dy}{dx} + x\frac{dy}{dx} + y \times 1 + \cos x + e^y \frac{dy}{dx}$

Sol: $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

$\frac{dy}{dx} = \frac{\cos(cx+d)\cos(ax+b)\cdot a + \sin(ax+b)\cdot \sin(cx+d)\cdot c}{\cos^2(cx+d)}$

$= \frac{\cancel{\cos(cx+a)} \cdot \cos(ax+b)\cdot a}{\cos^{\cancel{2}}(cx+d)} + \frac{\sin(ax+b)\cdot \sin(cx+d)\, c}{\cos^2(cx+d)}$

$= \frac{1}{\cos(cx+a)} \cdot \cos(ax+b)\, a + c\cdot \sin(ax+b) \cdot \tan(cx+a) \cdot \sec(cx+a)$

$= a\cos(ax+b) \cdot \sec(cx+d) + c \cdot \sin(ax+b) \cdot \tan(cx+d) \cdot \sec(cx+d)$

derivative of Implicit functions

$y = x^2 + \sin x + e^y$ ⟶ find $\frac{dy}{dx}$

Qus:

$$y = x^2 + \sin x + e^y$$

find $\frac{dy}{dx}$ $\longleftrightarrow$ find $\frac{dx}{dy}$

$\square^3$
$2\square^{2} \cdot \square$

D.w.r.t. 'x'

$$2y\frac{dy}{dx} = 2x + \cos x + e^y \cdot \frac{dy}{dx}$$

$$2y\frac{dy}{dx} - e^y\frac{dy}{dx} = 2x + \cos x$$

$$\frac{dy}{dx}(2y - e^y) = 2x + \cos x$$

$$\boxed{\frac{dy}{dx} = \frac{2x + \cos x}{2y - e^y}}$$

$$\frac{dx}{dy} = \frac{2y - e^y}{2x + \cos x}$$

Ex– $x^3 + y^3 + xy + y^2 + \log x = e^{2y}$ find $\frac{dy}{dx}$

D.w.r.t. 'x'

$$3x^2 + \boxed{3y^2\frac{dy}{dx}} + \boxed{x\frac{dy}{dx}} + y + \boxed{2y\frac{dy}{dx}} + \frac{1}{x} = \boxed{e^{2y} \cdot 2\frac{dy}{dx}}$$

$$3y^2\frac{dy}{dx} + x\frac{dy}{dx} + 2y\frac{dy}{dx} - 2e^{2y}\frac{dy}{dx} = -\frac{1}{x} - y - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 + x + 2y - 2e^{2y}\right) = -\frac{(1 + xy + 3x^3)}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{-(1 + xy + 3x^2)}{x(3y^2 + x + 2y - 2e^{2y})}}$$

Ex– $\sin(xy) = \log\sqrt{xy} - e^y + \sin x$ find $\boxed{\frac{dy}{dx}}$ → w.r.t. 'x'

$$\cos(xy)\left[x\frac{dy}{dx} + y\right] = \frac{1}{\sqrt{xy}} \cdot \frac{1}{2\sqrt{xy}}\left[x\frac{dy}{dx} + y\right] - e^y\frac{dy}{dx} + \cos x$$

$$x\cos(xy)\frac{dy}{dx} + y \cdot \cos(xy) = \frac{1}{2xy}\left[x\frac{dy}{dx} + y\right] - e^y\frac{dy}{dx} + \cos x$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = \frac{x}{2xy}\frac{dy}{dx} + \frac{y}{2xy} - e^y\frac{dy}{dx} + \cos x$$

$$\boxed{x\cos(xy)\frac{dy}{dx}} + y\cos(xy) = \boxed{\frac{1}{2y}\frac{dy}{dx}} + \frac{1}{2x} - \boxed{e^y\frac{dy}{dx}} + \cos x$$

$$x\cos(xy)\frac{dy}{dx} - \frac{1}{2y}\frac{dy}{dx} + e^y\frac{dy}{dx} = \frac{1}{2x} + \cos x - y\cos(xy)$$

$$\frac{dy}{dx}\left[x\cos(xy) - \frac{1}{2y} + e^y\right] = \frac{1}{2x} + \cos x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{\frac{1}{2x} + \cos x - y\cos(xy)}{x\cos(xy) - \frac{1}{2y} + e^y}$$

Sol. $\sin^2 y + \cos xy = \ldots$

Sol. $\sin^2 y + \cos xy = \pi$

$\boxed{2\sin y \cdot \cos y}\frac{dy}{dx} - \sin xy\left(x\frac{dy}{dx} + y\right) = 0$

$\boxed{\sin 2y \frac{dy}{dx}} - \boxed{x \cdot \sin xy \frac{dy}{dx}} + y \sin xy = 0$

$\sin 2y \frac{dy}{dx} - x \sin xy \frac{dy}{dx} = -y \sin xy$

$\frac{dy}{dx}(\sin 2y - x \sin xy) = -y \sin xy$

$\frac{dy}{dx} = \frac{-y \sin xy}{\sin 2y - x \sin xy}$

Sol. $x^3 + x^2 y + xy^2 + y^3 = 81$

$3x^2 + x^2\frac{dy}{dx} + y \cdot 2x + x \cdot 2y\frac{dy}{dx} + y^2 \times 1 + 3y^2\frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} +$

<u>derivative of Inverse Trigo.</u>

ILATE

Inv. Tr.

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$ $\quad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad \frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$

Ex- If $y = \sin^{-1}\sqrt{x} + \cos^{-1}x^2$ find $\frac{dy}{dx}$

D. w. r. t. 'x'

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{1-x^4}} \cdot 2x$

EXERCISE 5.4

Differentiate the following w.r.t. x:

1. $\frac{e^x}{\sin x}$ 2. $e^{\sin^{-1} x}$ 3. e^{x^3}

4. $\sin(\tan^{-1} e^{-x})$ 5. $\log(\cos e^x)$ 6. $e^x + e^{x^2} + ... + e^{x^5}$

7. $\sqrt{e^{\sqrt{x}}}, x > 0$ 8. $\log(\log x), x > 1$ 9. $\frac{\cos x}{\log x}, x > 0$

10. $\cos(\log x + e^x), x > 0$

$y = \sqrt{e^{\sqrt{x}}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$

$\frac{e^{\sqrt{x}}}{2\sqrt{e^{\sqrt{x}}}} \cdot 2\sqrt{x} = \frac{e^{\sqrt{y}}}{4\sqrt{x e^{\sqrt{y}}}}$

(1) $\sin 2x = 2\sin x \cdot \cos x$

$\sin 2x = \dfrac{2\tan x}{1+\tan^2 x}$

(2) $\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

(3) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

7. $\sin^2 y + \cos xy = \pi$ **8.** $\sin^2 x + \cos^2 y = 1$ **9.** $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

10. $y = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right),\ -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}}$

11. $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), 0 < x < 1$

12. $y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), 0 < x < 1$

13. $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right), -1 < x < 1$

14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

15. $y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right), 0 < x < \dfrac{1}{\sqrt{2}}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$

$\dfrac{\pi}{2} - \cos^{-1}x = \sin^{-1}x$ [redacted]

$\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}$

$\operatorname{cosec}^{-1}x + \sec^{-1}x = \dfrac{\pi}{2}$

Sol. 9 $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

put, $x = \tan\theta$

$y = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

D. w. r. t. 'x'

$\dfrac{dy}{dx} = 2\cdot\left(\dfrac{1}{1+x^2}\right)$

$= \dfrac{2}{1+x^2}$

$y = \sin^{-1}(\cos 2\theta)$

$y = \dfrac{\pi}{2} - \cos^{-1}(\cos 2\theta)$

$y = \dfrac{\pi}{2} - 2\theta$

$y = \dfrac{\pi}{2} - 2\tan^{-1}x$

$\dfrac{dy}{dx} = 0 - 2\cdot\left(\dfrac{1}{1+x^2}\right) = \dfrac{-2}{1+x^2}$

Second order derivative

Ex- If $y = e^{2x}$ find $\dfrac{d^2y}{dx^2}$

$\dfrac{dy}{dx} = e^{2x}\cdot 2$

again d. w. r. t. 'x'

$\dfrac{d^2y}{dx^2} = e^{2x}\cdot 2\cdot 2$

$= 4\cdot e^{2x}$

FOD $= \dfrac{dy}{dx} = y_1 = y' = f'(x)$

SOD $= \dfrac{d^2y}{dx^2} = y_2 = y'' = f''(x)$

$$\frac{d^2y}{dx^2} = 4 \cdot e^{2x}$$

Ex- $y = \log(\log x)$ find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{1}{x \log x}$$

again D. w. r. t 'x'

$$\frac{d^2y}{dx^2} = \frac{x\log x \frac{d}{dx} 1 - 1 \frac{d}{dx}(x \log x)}{(x \log x)^2}$$

$$= \frac{x \log x \times 0 - 1\left(x \cdot \frac{1}{x} + \log x\right)}{(x \log x)^2}$$

$$= \frac{-(1 + \log x)}{(x \log x)^2}$$

Find the second order derivatives of the functions given in Exercises 1 to 10.

1. $x^2 + 3x + 2$
2. x^{20}
3. $x \cdot \cos x$
4. $\log x$
5. $x^3 \log x$
6. $e^x \sin 5x$
7. $e^{6x} \cos 3x$
8. $\tan^{-1} x$
9. $\log(\log x)$
10. $\sin(\log x)$
11. If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\frac{d^2y}{dx^2} = \frac{(1+x^2)\times 0 - 1(2x)}{(1+x^2)^2} =$$

$$-3e^{6x} \cdot \sin 3x + 6\cos 3x \cdot e^{6x}$$

$$-3(\quad) + 6(\quad)$$

$$-3\left(e^{6x} \cdot \cos 3x \cdot 3 + \sin 3x \cdot e^{6x} \cdot 6\right) + 6\left(\cos 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot (-\sin 3x) \cdot 3\right)$$

$$-3\left(3\cos 3x \cdot e^{6x} + 6 \sin 3x \cdot e^{6x}\right) + 6\left(6 \cdot \cos 3x \cdot e^{6x} - 3e^{6x} \cdot \sin 3x\right)$$

$$-9\cos 3x \cdot e^{6x} - 18 \sin 3x \cdot e^{6x} + 36 \cos 3x \cdot e^{6x} - 18 e^{6x} \cdot \sin 3x$$

$$27 \cos 3x \cdot e^{6x} - 36 \sin 3x \cdot e^{6x} = 9e^{6x}(3\cos 3x - 4 \sin 3x)$$

12. If $y = \cos^{-1} x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.
13. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$
14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$
15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$
16. If $e^y(x + 1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$
17. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$

$$y_1 = \frac{2\tan^{-1}x}{1+x^2}$$

Sol. 16

$$e^y (x+1) = 1$$

$$e^y(1+0) + (x+1)e^y \frac{dy}{dx} = 0$$

$$e^y + (x+1)e^y \frac{dy}{dx} = 0$$

$$e^y\left[1 + (x+1)\frac{dy}{dx}\right] = 0$$

$$1 + (x+1)\frac{dy}{dx} = 0$$

$$(x+1)\frac{dy}{dx} = -1$$

$$y_1 = \frac{2\tan^{-1}x}{1+x^2}$$

$$\underbrace{(1+x^2)}_{I}.\underbrace{y_1}_{II} = 2\tan^{-1}x$$

again D w.r.t. 'x'

$$(1+x^2)y_2 + y_1(0+2x) = \frac{2}{(1+x^2)}$$

$$(1+x^2)^2 y_2 + y_1 2x(1+x^2) = 2$$

$$(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

$$\ldots \frac{dy}{dx} = 0$$

$$(x+1)\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = \frac{-1}{x+1}$$

again D.w.r.t 'x'

$$\frac{d^2y}{dx^2} = \frac{(x+1)\times 0 - (-1)(1)}{(x+1)^2} = \frac{1}{(x+1)^2} = \left[\frac{-1}{(x+1)}\right]^2$$

$$\boxed{\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2}$$

12. If $y = \cos^{-1} x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$

14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

16. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

17. If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

$$\frac{dy}{dx} = 500e^{7x}\cdot 7 + 600e^{-7x}\cdot(-7)$$

$$\frac{dy}{dx} = 7\left(500e^{7x} - 600e^{-7x}\right)$$

again D.w.r.t. 'x'

Skew - symmetric Matrix

If $A' = -A$

then A is SSM

$$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$$

→ all elements should be zero

$$A' = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$$

$$A' = -A$$

Logarith. properties

1. $\log x^m = m\log x$
2. $\log(xy) = \log x + \log y$
3. $\log\left(\frac{x}{y}\right) = \log x - \log y$
4. $\log e = 1$

$(v_1)^{v_2}$

Let $y = x^y$ → derivative $\frac{dy}{dx}$

3. $\log\left(\frac{x}{y}\right) = \log x - \log y$

4. $\log e = 1$

5. $\log 1 = 0$

Ex. If $y = x^x$ find $\frac{dy}{dx}$

$\log y = \log x^x$

$\log y = x \cdot \log x$

D.w.r.t. 'x'

$\frac{1}{y}\frac{dy}{dx} = \cancel{x} \cdot \frac{1}{\cancel{x}} + \log x \times 1$

$\frac{dy}{dx} = y\,(1 + \log x)$

$= x^x(1+\log x)$

Let $y = x^y \rightarrow$ derivative $\frac{dy}{dx}$

$y = x^y$

Taking log on both side

$\log y = \log x^y$

$\log y = y \cdot \log x$

D.w.r.t. 'x'

$\frac{1}{y}\frac{dy}{dx} = y \cdot \frac{1}{x} + \log x \frac{dy}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{y}{x} + \log x \frac{dy}{dx}$

$\frac{1}{y}\frac{dy}{dx} - \log x \frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

$$\boxed{\frac{dy}{dx} = \frac{\frac{y}{x}}{\frac{1}{y} - \log x}}$$

Ex. If $y^x = x^y$ find $\frac{dy}{dx}$

Taking log on both side

$\log y^x = \log x^y$

$x \cdot \log y = y \cdot \log x$

D.w.r.t. 'x'

$x \cdot \frac{1}{y}\frac{dy}{dx} + \log y \cdot 1 = y \cdot \frac{1}{x} + \log x \frac{dy}{dx}$

$\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \log x \frac{dy}{dx}$

$\frac{x}{y}\frac{dy}{dx} - \log x \frac{dy}{dx} = \frac{y}{x} - \log y$

$$\frac{dy}{dx} = \frac{\frac{y}{x} - \log y}{\frac{x}{y} - \log x} = \frac{\frac{y - x\log y}{x}}{\frac{x - y\log x}{y}} = \frac{y\,(y - x\log y)}{x\,(x - y\log x)}$$

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

1. $\cos x \,.\, \cos 2x \,.\, \cos 3x$

2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

3. $(\log x)^{\cos x}$

4. $x^x - 2^{\sin x}$

5. $(x+3)^2 \,.\, (x+4)^3 \,.\, (x+5)^4$

6. $\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

$y = \cos x \cdot \cos 2x \cdot \cos 3x$

$\log(xy) = \log x + \log y$

$\log y = \log(\cos x \cdot \cos 2x \;\; \cos 3x)$

$\log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$

D.w.r.t. 'x'

$$\log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$$

D. w. r. t. 'x'

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) + \frac{1}{\cos 2x}(-\sin 2x)\,2 + \frac{1}{\cos 3x}(-\sin 3x)\cdot 3$$

$$\frac{1}{y}\frac{dy}{dx} = -\tan x - 2\tan 2x - 3\tan 3x$$

$$\frac{dy}{dx} = -y(\tan x + 2\tan 2x + 3\tan 3x) = \underline{-\cos x\cdot\cos 2x\cdot\cos 3x(\tan x + 2\tan 2x + 3\tan 3x)}$$ Ans

$$y = (\log x)^{\cos x}$$

$$\log y = \log(\log x)^{\cos x}$$

$$\log y = \underline{\cos x}\cdot\underline{\log(\log x)}$$

D. w. r. t. '(x)'

$$\frac{1}{y}\frac{dy}{dx} = \cos x\cdot\frac{1}{x\cdot\log x} + \log(\log x)\cdot(-\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x\log x} - \sin x\cdot\log(\log x)$$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \sin x\cdot\log(\log x)\right]$$

$$= (\log x)^{\cos x}\left(\frac{\cos x}{x\log x} - \sin x\,\log(\log x)\right)$$

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

1. $\cos x \,.\, \cos 2x \,.\, \cos 3x$
2. $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$
3. $(\log x)^{\cos x}$
4. $x^x - 2^{\sin x}$
5. $(x+3)^2 \,.\, (x+4)^3 \,.\, (x+5)^4$
6. $\left(x+\dfrac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

$$y = \sqrt{}$$

$$y = \left[\frac{(x-1)\;(x-2)}{(x-3)(x-4)(x-5)}\right]^{\frac{1}{2}}$$

$$\log y = \log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]^{\frac{1}{2}}$$

$$\log\left(\frac{x}{y}\right) =$$

$$\log y = \frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]$$

$$\log y = \frac{1}{2}\left[\log(x-1)(x-2) - \log(x-3)(x-4)(x-5)\right]$$

$$\log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) - (\log(x-3) + \log(x-4) + \log(x-5))\right]$$

$$\log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)\right]$$

D. w. r. t. 'x'

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

$$\frac{dy}{dx} = \frac{y}{2}\left(\right)$$

$$\frac{dy}{dx} = \frac{y}{2}\left(\qquad \right)$$

$$= \frac{\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

1. $\cos x \,.\, \cos 2x \,.\, \cos 3x$
2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$
3. $(\log x)^{\cos x}$
4. $x^x - 2^{\sin x}$
5. $(x+3)^2 \,.\, (x+4)^3 \,.\, (x+5)^4$
6. $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Let; $y = (x+3)^2(x+4)^3(x+5)^4$

Taking Log on both side

$$\log y = \log\left[(x+3)^2(x+4)^3(x+5)^4\right]$$

$$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$$

$$\log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$$

D.w.r.t. 'x'

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right] = (x+3)^2(x+4)^3(x+5)^4\left[\frac{2}{x+}\right.$$

Q. $y = x^x + y^x + x^y$

Let u, v, w

$$y = u + v + w$$

D.w.r.t. 'x'

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \quad \text{——} \quad (1)$$

$$u = x^x$$

$$\log u = \log x^x$$

$$\log u = x.\log x$$

$$\frac{1}{u}\frac{du}{dx} = \frac{x}{x} + \log x$$

$$\frac{du}{dx} = u(1+\log x)$$

$$\frac{du}{dx} = x^x(1+\log x)$$

$$v = y^x$$

$$\log v = \log y^x$$

$$\log v = x.\log y$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$$

$$\frac{dv}{dx} = v\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

$$= y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

$$w = x^y$$

$$\log w = \log x^y$$

$$\log w = y\cdot\log x$$

$$\frac{1}{w}\frac{dw}{dx} = \frac{y}{x} + \log x\cdot\frac{dy}{dx}$$

$$\frac{dw}{dx} = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$$

from (1)

$$\frac{dy}{dx} = x^x(1+\log x) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$$

$$\boxed{\frac{dy}{dx}} = x^x(1+\log x) + \boxed{\frac{y^x.x}{y}\frac{dy}{dx}} + y^x.\log y + \frac{x^y.y}{x} + \boxed{x^y.\log x\frac{dy}{dx}}$$

$$\frac{dy}{dx}\left(1 - \frac{y^x.x}{y} - x^y\log x\right) = x^x(1+\log x) + y^x\log y + \frac{x^y y}{x}$$

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

1. $\cos x \cdot \cos 2x \cdot \cos 3x$
2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$
3. $(\log x)^{\cos x}$
4. $x^x - 2^{\sin x}$
5. $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$
6. $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

$y = u - w$

$\frac{dy}{dx} = \frac{du}{dx} - \boxed{\frac{dw}{dx}}$

$w = 2^{\sin x}$

$\log w = \log 2^{\sin x}$

$\log w = \sin x \cdot \log 2$

$\frac{1}{w}\frac{dw}{dx} = \sin x \cdot \frac{1}{2} \cdot 0 + \log 2 \cdot \cos x$

$\frac{1}{w}\frac{dw}{dx} = \log 2 \cdot \cos x$

$\frac{dw}{dx} = w \log 2 \cdot \cos x$

$= 2^{\sin x} \cdot \log 2 \cdot \cos x$

⑥ $y = \left(x+\frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ — (1)

$u = \left(x+\frac{1}{x}\right)^x$

$\log u = \log\left(x+\frac{1}{x}\right)^x$

$\log u = x \cdot \log\left(x+\frac{1}{x}\right)$

$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{\frac{x}{1}+\frac{1}{x}} \cdot \left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right) \cdot 1$

$= \frac{\cancel{x} \cdot x}{x^2+1}\left(\frac{x^2-1}{\cancel{x^2}}\right) + \log\left(x+\frac{1}{x}\right)$

$= \frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)$

$\frac{du}{dx} = u\left[\quad\right] = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right]$

$\frac{d}{dx}\frac{1}{x^n} = \frac{-n}{x^{n+1}}$

$v = x^{\left(x+\frac{1}{x}\right)}$

$\log v = \log x^{\left(x+\frac{1}{x}\right)}$

$\log v = \left(x+\frac{1}{x}\right) \cdot \log x$

$\frac{1}{v}\frac{dv}{dx} = \left(x+\frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(1-\frac{1}{x^2}\right)$

$\frac{dv}{dx} = v\left[\left(x+\frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(1-\frac{1}{x^2}\right)\right]$

$= x^{\left(x+\frac{1}{x}\right)}\left[\left(x+\frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(1-\frac{1}{x^2}\right)\right]$

+

Q. $y = (\log x)^x + x^{\log x}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ — (1)

$u = (\log x)^x \longrightarrow \cancel{x \log x} \longrightarrow \log(x)^x$

$\log u = \log((\log x)^x)$

$v = x^{\log x}$

$\log v = \log x^{\log x}$

$\log v = \log x \cdot \log x = (\log x)^2$

$\frac{1}{v}\frac{dv}{dx} = \frac{2\log x}{x}$

$\frac{dv}{dx} = v\left(\frac{2\log x}{x}\right) = x^{\log x}\left(\frac{2\log x}{x}\right)$

$$\frac{dv}{dx} = v\left(\frac{2\log x}{x}\right) = x \ldots \left(\frac{2\log x}{x}\right)$$

$$\log u = \log(\log x)^x$$

$$\log u = x \cdot \log(\log x)$$

$$\frac{1}{u}\frac{du}{dx} = \cancel{x}\cdot\frac{1}{\log x}\cdot\frac{1}{\cancel{x}} + \log(\log x)\cdot 1$$

$$\frac{1}{u}\frac{du}{dx} = \frac{1}{\log x} + \log(\log x)$$

$$\frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right]$$

(14) $$(\cos x)^y = (\cos y)^x$$

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y\cdot\log(\cos x) = x\cdot\log(\cos y)$$

D.w.r.t. 'x'

$$y\cdot\frac{1}{\cos x}\cdot(-\sin x) + \log(\cos x)\cdot\frac{dy}{dx} = x\cdot\frac{1}{\cos y}\cdot(-\sin y)\cdot\frac{dy}{dx} + \log(\cos y)\cdot 1$$

$$-y\tan x + \underbrace{\log(\cos x)\cdot\frac{dy}{dx}} = \underbrace{-x\cdot\tan y\,\frac{dy}{dx}} + \log(\cos y)$$

$$\log(\cos x)\frac{dy}{dx} + x\cdot\tan y\,\frac{dy}{dx} = \log(\cos y) + y\tan x$$

$$\frac{dy}{dx}\left[\log(\cos x) + x\tan y\right] = \log(\cos y) + y\tan x$$

$$\frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$$

$$x\log y = y\cdot\log x$$

$$\frac{x}{y}\frac{dy}{dx} + \log y\cdot 1 = \frac{y}{x} + \log x\,\frac{dy}{dx}$$

Sol. 8 $$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$

$\downarrow$ u $\quad\quad$ $\downarrow$ v

$$y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{—(1)}$$

$$\frac{du}{dx} = (\sin x)^x\left[x\cot x + \log(\sin x)\right]$$

$$v = \sin^{-1}\sqrt{x}$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x(1-x)}}$$

Sol. 11 $$y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$$

$\downarrow$ u $\quad\quad$ $\downarrow$ v

$$y = u + v$$

$$v = (x\cdot\sin x)^{\frac{1}{x}}$$

$$\log v = \log(x\cdot\sin x)^{\frac{1}{x}}$$

$$\log v = \frac{1}{x}\cdot\log(x\cdot\sin x)$$

$$y = u + v$$
$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{—(1)}$$

$$u = (x\cos x)^x$$
$$\log u = \log (x\cos x)^x$$
$$\log u = x \cdot \log(x \cdot \cos x)$$
$$\log u = x \cdot (\log x + \log \cos x)$$
$$\log u = x \cdot \log x + x \cdot \log(\cos x)$$
$$\frac{1}{u}\frac{du}{dx} = \frac{x}{x} + \log x \cdot 1 + x \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x) \cdot 1$$
$$\frac{1}{u}\frac{du}{dx} = 1 + \log x - x \cdot \tan x + \log(\cos x)$$
$$\frac{dy}{dx} = (x\cos x)^x \left[1 + \log x - x \cdot \tan x + \log(\cos x)\right]$$

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n}{x^{n+1}}$$

$$\log v = \log(x \cdot \sin x)^{\frac{1}{x}}$$
$$\log v = \frac{1}{x} \cdot \log(x \cdot \sin x)$$
$$\frac{1}{v}\frac{dv}{dx} = \frac{1}{x} \cdot \frac{1}{x\sin x}(x\cos x + \sin x \cdot 1) + \log(x \sin x) \cdot \left(\frac{-1}{x^2}\right)$$
$$\frac{1}{v}\frac{dv}{dx} = \frac{\cancel{x}\cos x}{x^{\cancel{2}}\sin x} + \frac{\cancel{\sin x}}{x^2 \cancel{\sin x}} - \frac{1}{x^2}\log(x\sin x)$$
$$\frac{1}{v}\frac{dv}{dx} = \frac{\cos x}{x} + \frac{1}{x^2} - \frac{1}{x^2}\log(x\sin x)$$
$$\frac{dv}{dx} = v\left[\frac{\cos x}{x} + \frac{1}{x^2} - \frac{1}{x^2}\log(x\sin x)\right]$$
$$\frac{dv}{dx} = (x \cdot \sin x)^{\frac{1}{x}}\left[\frac{\cos x}{x} + \frac{1}{x^2} - \frac{1}{x^2}\log(x\sin x)\right]$$

Sol.

$$xy = e^{x-y}$$
$$\log(xy) = \log e^{x-y}$$
$$\log x + \log y = x - y$$
D.w.r.t. 'x'
$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\frac{dy}{dx}\left(\frac{1}{y} + 1\right) = 1 - \frac{1}{x}$$
$$\frac{dy}{dx}\left(\frac{1+y}{y}\right) = \frac{x-1}{x}$$
$$\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$$

Q $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ — Ans: 120

$f'(1)$

$$f(1) = 2 \times 2 \times 2 \times 2 = 16$$
$$\log[f(x)] = \log\left[(1+x)(1+x^2)(1+x^4)(1+x^8)\right]$$
$$\log\boxed{f(x)} = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$
$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{1+x} + \frac{1}{1+x^2}2x + \frac{1}{1+x^4}4x^3 + \frac{1}{1+x^8} \cdot 8x^7$$
$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}$$
$$f'(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$
$$f'(1) = f(1)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$
$$f'(1) = \boxed{f(1)}\left[\frac{15}{2}\right]$$
$$= \frac{\cancel{16}^{8} \times 15}{\cancel{2}} = 8 \times 15 = 120$$

"Derivative of function in Parametric form"

$$x = \sin\theta$$
D.w.r.t. 'θ'
$$\frac{dx}{d\theta} = \cos\theta \quad \text{—(1)}$$

$$y = \cos\theta$$
D.w.r.t. 'x'
$$\frac{dy}{d\theta} = -\sin\theta \quad \text{—(2)}$$

$$\frac{dy}{dx} = ?$$

$\frac{dx}{d\theta} = \cos\theta$ — (1) $\qquad \frac{dy}{d\theta} = -\sin\theta$ — (2)

(2) ÷ (1)

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

$$\boxed{\frac{dy}{dx} = -\tan\theta}$$

Ex If $x = t^3$ $y = 4t^2$ find $\frac{dy}{dx}$

EXERCISE 5.6

If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

1. $x = 2at^2$, $y = at^4$
2. $x = a\cos\theta$, $y = b\cos\theta$
3. $x = \sin t$, $y = \cos 2t$
4. $x = 4t$, $y = \frac{4}{t}$
5. $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$
6. $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$
7. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$
8. $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ $y = a\sin t$
9. $x = a\sec\theta$, $y = b\tan\theta$
10. $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$
11. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta \qquad \frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

$\frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$

$\frac{dy}{dx} =$

$x = a(\cos\theta + \theta\sin\theta) \qquad y = a(\sin\theta - \theta\cos\theta)$

$\frac{dx}{d\theta} = a[-\cancel{\sin\theta} + \theta\cos\theta + \cancel{\sin\theta}]$

$\frac{dx}{d\theta} = a\theta\cos\theta$

$\frac{dy}{d\theta} = a[\cos\theta - (-\theta\sin\theta + \cos\theta)]$

$= a[\cancel{\cos\theta} + \theta\sin\theta - \cancel{\cos\theta}]$

$= \cancel{a\theta\cos\theta}\ a\theta\sin\theta$

$\boxed{\frac{dy}{dx} = \tan\theta}$

$\frac{dx}{dt} = a\left[-\sin t + \cot\frac{t}{2}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{\cancel{\cos\frac{t}{2}}}{\sin\frac{t}{2}}\cdot\frac{1}{\cos^2\frac{t}{2}}\cdot\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}\right]$ $\qquad \sin 2x = 2\sin x\cdot\cos x$

$= a\left[-\sin t + \frac{1}{\sin t}\right] = a\left[\frac{-\sin^2 t + 1}{\sin t}\right] = \frac{a\cos^2 t}{\sin t}$ — (1)

$\frac{dy}{dt} = a\cos t$ — (2)

$\frac{dy}{dx} = \frac{a\cos t}{\frac{a\cos^2 t}{\sin t}} = \cancel{a}\cancel{\cos t} \times \frac{\sin t}{\cancel{a}\cos^{\cancel{2}} t}$

$= \tan t$